Topology – II

Answer all question. Give complete justifications.

Max. Marks : 40

Time – 2 hours

1. State whether the following statements are TRUE or FALSE. Justify.

   (a) The map $\exp : [0, \infty) \to S^1$, $t \mapsto (\cos 2\pi t, \sin 2\pi t)$ is a covering map.

   (b) If X is path connected and locally path connected and $p : E \to X$ is a connected covering with $G(E/X) = \{1\}$, then p is a homeomorphism.

   (c) Finetely generated subgroups of a free group of rank 2 have finite index.

   (d) Every map $f : \mathbb{R}P^2 \to S^1 \times S^1$ is null homotopic.

   (e) If X and Y are connected spaces having homeomorphic universal covers, then X and Y are homeomorphic. [5×4]

2. Construct a covering of $S^1 \vee S^1$ corresponding to the normal subgroup of $\pi_1(S^1 \vee S^1)$ generated by a. Here 'a' has the usual meaning. [10]

3. Describe the connected coverings of $\mathbb{R}P^2 \times \mathbb{R}P^2$ up to covering space isomorphisms. [10].